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A Fully Implicit, Three-Dimensional, Two-Phase, Control-Volume Finite Element Model for the Simulation of Naturally Fractured Reservoirs

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Abstract

This paper describes the formulation and application of a fully implicit, three-dimensional, two-phase (water and oil) mathematical model for the simulation of naturally fractured reservoirs. The model equations are discretized using the control-volume finite element (CVFE) scheme where pressure terms are treated in a finite element manner while mobility terms are upstream-weighted as in the usual finite difference (FD) method. The form of the discretized equations is similar to the usual FD method, thus standard techniques are used to solve the Jacobian. The equations are solved fully implicit for pressure, saturation, and wellbore variables using the iterative Newton-Raphson method of solution.

Two field scale examples, that were first presented by Kazemi et al., were used to test the model. These include a quadrant of a five-spot pattern, and a five-well fractured reservoir having dip and natural water influx.

Introduction

Performance of naturally fractured reservoirs is complicated and more difficult to predict than conventional reservoirs due to the different mechanisms involved in the production of oil and gas. The complexity arises from the existence of two flow paths (matrix and fracture which have totally different properties) that communicate with each other.

Because of the irregularity of the fracture morphology, distribution, orientation, and extension, some sort of idealization of these reservoirs is necessary for the simulation purposes.

Kazemi et al. (1976) set the foundation for field-scale, dual-porosity, multi-phase simulators. In their idealization, an elemental reservoir volume in a naturally fractured reservoir (Fig. 1) is represented by a set of grid cells. Each grid cell may contain one or several matrix blocks. Accordingly, within any grid cell, all matrix blocks will have the same pressures and saturations. Similarly, the fractures within a grid cell will have identical pressures and saturations which differ from those of matrix blocks. A typical grid cell with its pressure distribution is shown in Fig. 2. This idealization was based on the following assumptions: (1) the matrix blocks are homogeneous and isotropic and form a uniform assemblage of identical, rectangular, parallelopipeds, (2) the fractures are contained within an orthogonal system of continuous, uniform, and constant-width paths where the fractures are parallel to the principal axes of permeability., (3) the flow takes place from fracture to fracture, from fracture to matrix, and from matrix to fracture, (4) the matrix blocks have high storage volume and low flow capacity whereas fractures have low storage volume and high flow capacity, and (5) the wellbore intersects the fracture network and hence the fractures provide the main path for fluid flow into the wellbore.

In many practical problems, this idealization is sufficient. In cases where a matrix block contains several grid nodes, the gravity segregation within the matrix block is calculated. This description points to the critical engineering decision in the selection of the number of grid nodes with respect to the number of matrix blocks.

Development of the Model Equations

In this work, the simulator to be discussed represents a single region of a naturally fractured reservoir. In formulating the fluid flow equations for this region, the following assumptions were implicitly made: (1) the continuity equation is used to describe the mass conservation in a control volume, (2) Darcy's law is used to describe fluid flow through fractures, and from matrix blocks to fractures, and (3) at the end of a time step, the reservoir system is assumed to be in a complete gravity-capillary equilibrium.

1. Fracture Flow Equation

$$\nabla \cdot \left[(k\phi)_f \left(\frac{k_r}{\mu B} \right)_{af} (\nabla p_{af} - \gamma_{af} \nabla D_f) \right] + \frac{\hat{t}_{a\text{ }mf}}{B_a} + \frac{\hat{q}_{af}}{B_a} = \frac{\partial}{\partial t} \left(\frac{\phi S_a}{B_a} \right)_f \quad (1)$$

2. Matrix/Fracture Flow Equation

$$k_m \left(\frac{k_r}{\mu B} \right)_{amf} \sigma [\nabla p_{amf} - \nabla(\gamma_a D)_{mf}] = \frac{\partial}{\partial t} \left(\frac{\phi S_a}{B_a} \right)_m \quad (2)$$

3. Wellbore Rate Equation

$$Q_T = \sum_a \left\{ \sum_{k=1}^{KK} q_{ak} + c_a \frac{\partial p_{ph}}{\partial t} \right\} \quad (3)$$

4. Initial Conditions

Initially, the reservoir should be in a gravity-capillary pressure equilibrium. In other words, the phase velocity in the vertical direction is zero. For fracture initialization, this condition is expressed mathematically as:

$$\left[(k_z \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \left(\frac{\partial p_{af}}{\partial z} - \gamma_{af} \frac{\partial D_f}{\partial z} \right) \right]_{t=0} = 0 \quad (4)$$

For matrix initialization, there would be no flow between the fracture and the matrix at equilibrium condition; i.e.

$$\left[k_m \left(\frac{k_r}{\mu B} \right)_{amf} \sigma (\nabla p_{amf} - \nabla(\gamma_a D)_{mf}) \right]_{t=0} = 0 \quad (5)$$

5. Boundary Conditions

For times greater than zero, boundary conditions must be specified. Four different types of boundary conditions are encountered in reservoir engineering applications. These are:

5.1. Constant Influx Boundary

$$\left[(k_\xi \phi)_f A \left(\frac{k_r}{\mu B} \right)_{af} \left(\frac{\partial p_{af}}{\partial \xi} - \gamma_{af} \frac{\partial D_f}{\partial \xi} \right) \right]_{\xi=b} = C \quad (6)$$

5.2. Constant Pressure Boundary

$$[p_{af}]_{\xi=b} = C \quad (7)$$

5.3. No-Flow Boundary

$$\left[(k_\xi \phi)_f A \left(\frac{k_r}{\mu B} \right)_{af} \left(\frac{\partial p_{af}}{\partial \xi} - \gamma_{af} \frac{\partial D_f}{\partial \xi} \right) \right]_{\xi=b} = 0 \quad (8)$$

5.4. Aquifer Boundary

$$\left[(k_\xi \phi)_f A \left(\frac{k_r}{\mu B} \right)_{af} \left(\frac{\partial p_{af}}{\partial \xi} - \gamma_{af} \frac{\partial D_f}{\partial \xi} \right) \right]_{\xi=b} = q_{aq} \quad (9)$$

6. Auxiliary Equations

In general, there are nine unknowns to be evaluated at each node: four fracture unknowns,

$$p_{wf}, p_{of}, S_{wf}, S_{of}$$

four matrix unknowns,

$$p_{wm}, p_{om}, S_{wm}, S_{om}$$

and one wellbore unknown,

$$p_{bh}$$

for the rate control well boundary condition.

The equations available so far are: two fracture equations represented by equation (1), two matrix/fracture equations represented by equation (2), and one well bore equation represented by equation (3). The other four auxiliary equations are:

$$\begin{aligned} p_{of} &= p_{wf} + P_{cf} \\ S_{of} &= 1 - S_{wf} \\ p_{om} &= p_{wm} + P_{cm} \\ S_{om} &= 1 - S_{wm} \end{aligned} \quad (10)$$

The above equations eliminate four unknowns. Thus, only five unknowns need to be evaluated at each node. These are:

$$p_{wf}, S_{wf}, p_{wm}, S_{wm}, \text{ and } p_{bh}$$

The CVFE Discretization Scheme

In the process of petroleum reservoir simulation, it is desirable to adopt the grid flexibility of the finite element method while retaining the phase upstream weighting of the usual FD methods. Phase upstream weighting is used for mobility terms since it often converges to the physically correct solution. It is difficult, however, to combine the upstream weighting with the usual finite element method for multiphase multidimensional flow problems. Such methods are not mass-conservative. In this paper, the CVFE method, using *linear triangular elements* (Fig. 3), is used for discretizing the model equations.

The pressure is treated in a finite element manner, while mobility terms are upstream weighted as in the usual finite difference way. The control-volume surrounding node i is constructed by connecting the set of barycenters with the connection mid-points of all elements connected to node i , (Fig. 4). The CVFE scheme for the solution of the model equations uses the integral formulation of these equations. It results in a set of nonlinear equations similar to those obtained in the FD method. Therefore the method of solution using the Jacobian is the same for both methods.

1. Fracture Flow Equation

Expanding the divergence operator of the fracture flow equation, equation (1), for the two-dimensional case yields:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\Delta z (k_x \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial x} \Phi_{af} \right] \\ & + \frac{\partial}{\partial y} \left[\Delta z (k_y \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial y} \Phi_{af} \right] \\ & + \Delta z \frac{\hat{\tau}_{amf}}{B_a} + \Delta z \frac{\hat{q}_{af}}{B_a} = \Delta z \frac{\partial}{\partial t} \left(\frac{\phi S_a}{B_a} \right)_f \end{aligned} \quad (11)$$

The CVFE discretization scheme starts with multiplying the above equation by a test function, $v(x,y)$, and integrating over the edges of the control volume, see Fig. 4:

$$\begin{aligned} & \int_{\Omega^e} \left\{ \frac{\partial}{\partial x} \left[\Delta z (k_x \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial x} \Phi_{af} \right] \right. \\ & \left. + \frac{\partial}{\partial y} \left[\Delta z (k_y \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial y} \Phi_{af} \right] \right\} v \, d\Omega^e \\ & + (V_R)_i \frac{\hat{\tau}_{amf}}{B_a} + (V_R)_i \frac{\hat{q}_{amf}}{B_a} = (V_R)_i \frac{\partial}{\partial t} \left(\frac{\phi S_a}{B_a} \right)_f \end{aligned} \quad (12)$$

where $(V_R)_i$ is the volume of the box surrounding node i . Thus we write:

$$\begin{aligned} & \int_{\Omega^e} \left\{ \frac{\partial}{\partial x} \left[\Delta z (k_x \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial x} \Phi_{af} \right] \right. \\ & \left. + \frac{\partial}{\partial y} \left[\Delta z (k_y \phi)_f \left(\frac{k_r}{\mu B} \right)_{af} \frac{\partial}{\partial y} \Phi_{af} \right] \right\} v \, d\Omega^e \\ & + (\tau_{amf})_i + (q_{af})_i = (V_R)_i \frac{\partial}{\partial t} \left(\frac{\phi S_a}{B_a} \right)_f \end{aligned} \quad (13)$$

The discretization process of the first term on the left-hand side is carried out by transferring differentiation from the original function to the test function by performing integration by parts (divergence theorem):

$$\begin{aligned}
& - \int_{\Omega^e} \left\{ \Delta z \left[(k_x \phi)_f \frac{\partial \Phi_{af}}{\partial x} \frac{\partial v}{\partial x} \right. \right. \\
& \quad \left. \left. + (k_y \phi)_f \frac{\partial \Phi_{af}}{\partial y} \frac{\partial v}{\partial y} \right] \left(\frac{k_r}{\mu B} \right)_{af} \right\} d\Omega^e \\
& + \oint_{\Gamma^e} \left\{ \Delta z \left[(k_x \phi)_f \frac{\partial \Phi_{af}}{\partial x} v n_x \right. \right. \\
& \quad \left. \left. + (k_y \phi)_f \frac{\partial \Phi_{af}}{\partial y} v n_y \right] \left(\frac{k_r}{\mu B} \right)_{af} \right\} d\Gamma^e
\end{aligned} \quad (14)$$

The boundary integral of the above equation disappears upon the assembly of the global stiffness matrix as the fluxes across element boundaries cancel one another. The variational approximation to the above variational form is performed by selecting:

$$\Phi(x, y, t) \approx \sum_j \Phi_j(t) \Psi_j(x, y) \quad (15)$$

$$v(x, y) \approx \Psi_i(x, y)$$

By referring to Fig. 3, it is obvious that the connection between nodes i and j will include contributions from the two elements sharing the common edge. Thus, for the control-volume of node i , equation (14) can be written as follows:

$$\begin{aligned}
& \sum_{j \in \eta_i} \left\{ - \sum_{e=1}^2 \int_{\Omega^e} \left[(k_x \phi)_{f,ij} \frac{\partial \Psi_i}{\partial x} \frac{\partial \Psi_j}{\partial x} \right. \right. \\
& \quad \left. \left. + (k_y \phi)_{f,ij} \frac{\partial \Psi_i}{\partial y} \frac{\partial \Psi_j}{\partial y} \right] d\Omega^e \right\} \Delta z_{ij} \left(\frac{k_r}{\mu B} \right)_{af,ij} \Phi_{ij}^{n+1}
\end{aligned} \quad (16)$$

where:

$$\begin{aligned}
\Phi_{ij} &= \Phi_j - \Phi_i = (p_j - p_i) - \gamma_{ij}(D_j - D_i) \\
\eta_i &= \text{set of all nodes surrounding node } i
\end{aligned} \quad (17)$$

Introducing transmissibility terms and rearranging yields:

$$\sum_{j \in \eta_i} (T)_{af,ij}^{n+1} \left[(p_{af,j}^{n+1} - p_{af,i}^{n+1}) - \gamma_{af,ij} (D_{f,j} - D_{f,i}) \right] \quad (18)$$

Since the third dimension is obtained by vertical projection of the two-dimensional case, therefore we simply add terms for the third dimension. The global equation for each control-volume is, then, written as follows:

$$\begin{aligned}
& \sum_{j \in \eta_i} (T)_{af,ij}^{n+1} \left[(p_{af,j}^{n+1} - p_{af,i}^{n+1}) - \gamma_{af,ij} (D_{f,j} - D_{f,i}) \right] \Big|_k \\
& + (T_z)_{af,k+\frac{1}{2}}^{n+1} \left[(p_{af,k+1}^{n+1} - p_{af,k}^{n+1}) - \gamma_{af,k+\frac{1}{2}} (D_{f,k+1} - D_{f,k}) \right] \\
& + (T_z)_{af,k-\frac{1}{2}}^{n+1} \left[(p_{af,k}^{n+1} - p_{af,k-1}^{n+1}) - \gamma_{af,k-\frac{1}{2}} (D_{f,k} - D_{f,k-1}) \right] \\
& + (\tau_{amf})_{ik}^{n+1} + (q_{af})_{ik}^{n+1} - \frac{(V_R)_{ik}}{5.6146 \Delta t} \left[\left(\frac{\phi S_a}{B_a} \right)^{n+1} \right. \\
& \quad \left. - \left(\frac{\phi S_a}{B_a} \right)^n \right]_{f,ik} = R_{af,ik}^{n+1}(\vec{X})
\end{aligned} \quad (19)$$

2. Matrix/Fracture Flow Equation

$$\begin{aligned}
& T_{am,ik}^{n+1} \left[(p_{af}^{n+1} - \gamma_{af}^{n+1} D_f) - (p_{am}^{n+1} - \gamma_{am}^{n+1} D_m) \right]_{ik} \\
& - \frac{(V_R)_{ik}}{5.6146 \Delta t} \left[\left(\frac{\phi S_a}{B_a} \right)^{n+1} - \left(\frac{\phi S_a}{B_a} \right)^n \right]_{m,ik} = R_{am,ik}^{n+1}(\vec{X})
\end{aligned} \quad (20)$$

3. Wellbore Rate Equation

$$\begin{aligned}
& Q_T - \sum_a \left\{ \sum_{k=1}^{KK} q_{ak} + c_a \frac{\partial p_{bh}}{\partial t} \right\} \\
& = Q_T - \sum_{k=1}^{KK} (q_w + q_o)_k - (c_w + c_o) \frac{\partial p_{bh}}{\partial t} \\
& = Q_T - \sum_{k=1}^{KK} (q_w + q_o)_k - \frac{(c_w + c_o)}{\Delta t} (p_{bh}^{n+1} - p_{bh}^{n+1}) \\
& = R_{bh,i}^{n+1}(\vec{X})
\end{aligned} \quad (21)$$

Numerical Examples

Two field scale examples, that were first presented by Kazemi et al., are used to test the accuracy of the results and the efficiency of the model formulation. These include a

quadrant of a five-spot pattern, and a five-well fractured reservoir having dip and natural water influx.

The relative permeability and capillary pressure curves, for the two examples, are shown by figures 5 and 6. It should be noted that the relative permeability to oil and water in the fracture covers the full spectrum of saturations from 0 to 1. The relative permeability in the matrix, however, is restricted to the mobil saturation range ($S_w = 0.25$ to $S_w = 0.7$).

The matrix capillary pressure is generally much greater than the fracture capillary pressure. The fracture capillary pressure declines rapidly with increased water saturation. This sets up a pressure differential from the matrix to the fracture causing the oil to flow from matrix to fracture and water to imbibe into the matrix blocks. The capillary pressure endpoints, however, in both matrix and fracture are the same, so the transition zones are identical in the fracture and the matrix. This is necessary, otherwise a static equilibrium for pressure and saturation distribution is not possible.

1. A Quadrant of a Five Spot Pattern

In this model problem, water is injected into one quarter of a developed conceptual five-spot pattern at a rate of 200 STB/D. Total liquid production rate is set at 210 STB/D. The reservoir is assumed to be fractured uniformly and was modeled using a 2D grid. Three different meshes, as shown by figures 7, 8, and 9, were used. Reservoir properties, for this model problem, are given in table 1. If reservoir parameters, however, are different from those assumed in the table, then recoveries will be substantially different from the results reported here.

Timesteps were calculated automatically using a maximum fracture saturation change of 0.01, and were restricted to a minimum of 0.01 day and a maximum of 10 days. A comparison of water/oil ratio from this work with that calculated by Kazemi et al. and Dutra and Aziz is shown in Fig. 10. Fig. 11 compares the movable oil pore volume water injected versus the movable oil pore volume oil produced for the three different meshes for different mobility ratios of 1, 10, and 50.

Additional runs using maximum timestep sizes of 20, 30, 40, 50, and 60 days were made to test the stability of the model. As it can be seen from Fig. 12, the results are identical for all selected time step sizes.

2. A Five-Well Fractured Reservoir Model

A conceptual five-well fractured reservoir model which is tilted along the x-direction at an angle of 5° is presented.

For this problem, Kazemi et al. used 13 blocks in the x-direction, 5 blocks in the y-direction, and 1 block in the z-direction. The block dimensions are 500x500x90 ft. Each reservoir block is further subdivided into 25 matrix blocks-100 ft on each side in the x and y directions. Fig. 13 shows the CVFE mesh used for this problem. The reservoir properties are given in table 2.

A total influx of 2,800 STB/D of water was assigned at the five nodes along the lowest part of the reservoir. Five production wells are arranged along the center-line in the x-direction as depicted by Fig. 13. Well 1 was initially producing at a total liquid rate of 1000 STB/D. It was shut in after 190 days when it went to a high WOR and well 4 was opened at the same rate. Well 2 was never produced because of its high water saturation. Wells 3 and 5 were producing at a total liquid rate of 1000 STB/D.

Timesteps were calculated automatically using a maximum fracture saturation change of 0.01. Timestep size was restricted to a minimum of 0.01 day and a maximum of 10 days. A comparison of water/oil ratio from this work with that calculated by Kazemi et al., for the four producing wells, is shown in Fig. 14.

Conclusions

1. The CVFE scheme allows the simulation of complex reservoir geometries in the areal plane more accurately, whereas the third dimension is modeled by vertical projection of the two-dimensional grid.
2. The CVFE method essentially gave identical results to the nine-point FD schemes, presented by Kazemi and Dutra and Aziz models, which is the most reliable solution for reducing the GOE.
3. The CVFE formulation of fluid flow in porous media, where the principal permeability tensor components are in the direction of coordinate axes, is practical. However, in situations where non-tensor permeability channelling does not coincide with the coordinate axes, it can still be accommodated in the CVFE grid.

Nomenclature

A	= area, ft ²
B	= formation volume factor, bbl/STB
c	= compressibility, 1/psi
CVFE	= control volume finite element
D	= depth, ft
FD	= finite difference
GOE	= grid orientation effect
h	= thickness, ft
k	= absolute permeability, md
k _r	= relative permeability, fraction

k_x	= x-component of the permeability tensor, md
k_y	= y-component of the permeability tensor, md
k_z	= z-component of the permeability tensor, md
M	= mobility ratio
p	= pressure, psi
p_c	= capillary pressure, psi
q	= volume flow rate, bbl/day
Q_T	= wellbore total flow rate, bbl/day
r	= radial distance, ft
r_l	= equivalent wellblock radius, ft
r_{ij}	= distance between nodes i and j, ft
r_w	= wellbore radius, ft
R	= residual of Newton-Raphson scheme
s	= skin factor
S	= saturation, fraction
t	= time, days
T	= transmissibility coefficient, bbl cp/(day psi)
V_R	= block volume, ft ³
w_{ij}	= width between nodes i and j, ft
WI	= well index, bbl cp/(day psi)
WOC	= water/oil contact
WOP	= oil-phase well productivity, STB/(day psi)
WOR	= water/oil ratio
WP	= well productivity=(WWP+WOP), STB/(day/psi)
WWP	= water-phase well productivity, STB/(day psi)

Greek Letters

α	= phase (water or oil)
β	= well code (0 for production and 1 for injection)
Γ^e	= boundary line of an element
γ	= fluid phase gradient, psi/ft
γ_{ij}	= entry in the global geometric stiffness matrix
η_i	= set of all surrounding nodes to node i
Δt	= time-step length, day
u	= viscosity, cp
v	= velocity, bbl/ft ²
ξ	= direction (x, y, or z)
ρ	= density, lb/ft ³
σ	= matrix block's shape factor, ft ²
ϕ	= porosity, fraction
Ω^e	= domain of an element

Subscripts

aq	= aquifer
b	= boundary
bh	= bottom hole
c	= capillary
c	= connate
CV	= control volume
f	= fracture
ij	= connection between nodes i and j
ik	= node i and layer k
m	= matrix
o, w	= oil, water
r	= relative
r	= residual
R	= reservoir

References

1. Brand, C.W., Z.E. Heinemann, and K. Aziz: "The Grid Orientation Effect in Reservoir Simulation," Paper SPE 21228 was Prepared for Presentation at the 11th SPE Symposium on Reservoir Simulation, Anaheim, CA (Feb. 17-20, 1991).
2. Dutra, T.V., and K. Aziz: "A New Double-Porosity Reservoir Model for Oil/Water Flow Problems," Paper SPE 21248 was Prepared for Presentation at the 11th SPE Symposium on Reservoir Simulation, Anaheim, CA (Feb. 17-20, 1991).
3. Forsyth, P.A.: "A Control-Volume, Finite-Element Method for Local Mesh Refinement in Thermal Reservoir Simulation," SPERE (Nov., 1990) 561-566.
4. Fung, L.S.K., A.D. Hiebert, and L. Nghiem: "Reservoir Simulation with a Control-Volume Finite-Element Method," Paper SPE 21224 was Prepared for Presentation at the 11th SPE Symposium on Reservoir Simulation, Anaheim, CA (Feb. 17-20, 1991).
5. Fung, L.S.K., W.L. Buchanan, and Ravi Sharma: "Hybrid-CVFE Method for Flexible Grid Reservoir Simulation," Paper SPE 25266 was Prepared for Presentation at the 12th SPE Symposium on Reservoir Simulation, New Orleans, LA (Feb. 28-March 3, 1993).
6. George, P.L.: "Automatic Mesh Generation-Application to the Finite Element Methods," Wiley Publishing Company, New York (1991).
7. IMSL: "MATH/LIBRARY: FORTRAN Subroutines for Mathematical Applications," Version 2.0, Houston, TX (1991).
8. Kazemi, H., L.S. Merrill, Jr., K.L. Porterfield and P.R. Zeman: "Numerical Simulation of Water-Oil Flow in Naturally Fractured Reservoirs," Paper SPE 5719 was First Presented at the SPE-AIME Fourth Symposium on Numerical Simulation of Reservoir Performance, Los Angeles, CA (Feb. 19-20, 1976).
9. Naji, H. S.: "Finite Difference and Control-Volume Finite Element Simulation of Naturally Fractured Reservoirs," PhD Dissertation, Petroleum Engineering Department, Colorado School of Mines, Golden, CO (Oct. 1993).
10. Ortega, J.M., and W.C. Rheinboldt: "Iterative Solution of Nonlinear Equations in Several Variables," Academic Press, Inc., Boston (1970).
11. Peaceman, D.W.: "Interpretation of Well-Block Pressures in Numerical Reservoir Simulation with Nonsquare

11. Peaceman, D.W.: "Interpretation of Well-Block Pressures in Numerical Reservoir Simulation with Nonsquare Grid Blocks and Anisotropic Permeability," SPEJ (Jun. 1983) 531-543.
12. Shinta, A.A.: "A Novel Numerical Simulation Approach for Characterization of Naturally Fractured Reservoirs," PhD Dissertation, Petroleum Engineering Department, Colorado School of Mines, Golden, CO (Feb. 1992).
13. Thomas, L.K., T.N. Dixon, and R.G. Pierson: "Fractured Reservoir Simulation," SPEJ (Feb. 1983) 42-54.
14. Warren, J.E., and P.J. Root: "The Behavior of Naturally Fractured Reservoirs," SPEJ (Sep. 1963) 245-255.

Appendix A

Fracture Transmissibilities

Consider the evaluation of transmissibility between nodes i and j , i.e. $(T)_{af,ij}$, where at least one of the nodes i or j is not on the external boundary. Mathematically, the transmissibility can be written as follows:

$$(T)_{af,ij}^{n+1} = 0.001127 \gamma_{ij} \left(\frac{A}{r} \right)_{ij} \left(\frac{k_r}{\mu B} \right)_{af,ij}^{n+1} \quad (22)$$

Where:

$$\begin{aligned} \gamma_{ij} &= - \sum_{\epsilon=1}^2 \int_{\Omega^{\epsilon}} \left[\left(\overline{k_x} \phi \right)_{f,ij} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} \right. \\ &\quad \left. + \left(\overline{k_y} \phi \right)_{f,ij} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right] d\Omega^{\epsilon} \\ &= - \sum_{\epsilon=1}^2 \left[\frac{\left(\overline{k_x} \phi \right)_{f,ij} \beta_i \beta_j + \left(\overline{k_y} \phi \right)_{f,ij} \gamma_i \gamma_j}{4A_{ij}} \right] \end{aligned} \quad (23)$$

$$\left(\overline{k_x} \phi \right)_{f,ij} = \frac{2 \left(\overline{k_x} \phi \right)_{f,i} \left(\overline{k_x} \phi \right)_{f,j}}{\left(\overline{k_x} \phi \right)_{f,i} + \left(\overline{k_x} \phi \right)_{f,j}} \quad (24)$$

$$\left(\overline{k_y} \phi \right)_{f,ij} = \frac{2 \left(\overline{k_y} \phi \right)_{f,i} \left(\overline{k_y} \phi \right)_{f,j}}{\left(\overline{k_y} \phi \right)_{f,i} + \left(\overline{k_y} \phi \right)_{f,j}} \quad (25)$$

$$\psi_i = \frac{1}{2A_{\epsilon}} (\alpha_i + \beta_i x + \gamma_i y)$$

$$\alpha_i = x_j y_k - x_k y_j$$

$$\beta_i = y_j - y_k \quad (26)$$

$$\gamma_i = x_k - x_j$$

$$A_{\epsilon} = \frac{1}{2} \sum_{i=1}^3 \alpha_i$$

$$x_{barycenter} = \sum_{i=1}^3 x_i / 3$$

$$y_{barycenter} = \sum_{i=1}^3 y_i / 3$$

$$A_{ij} = w_{ij} \Delta z_{ij}$$

$$w_{ij} \text{ is the width between nodes } i \text{ and } j \quad (27)$$

$$\Delta z_{ij} = \frac{(\Delta z_i + \Delta z_j)}{2}$$

$$r_{ij} \text{ is the distance between nodes } i \text{ and } j$$

$$i \neq j \neq k \text{ are permuted in a counter}$$

$$\text{clockwise direction (see Fig. 3)}$$

$$\left(\frac{k_r}{\mu B} \right)_{af,ij}^{n+1} = \left[\omega_{af,i} \left(\frac{k_r}{\mu B} \right)_{af,i}^{n+1} + (1 - \omega_{af,i}) \left(\frac{k_r}{\mu B} \right)_{af,j}^{n+1} \right] \quad (28)$$

The upstream weighting factor ω is calculated between nodes i and j and stored in i as $\omega_{af,i}$. ω has two values, either 1 or 0, obtained from the node of higher potential. The transmissibility is calculated between nodes i and j and stored in i as $T_{af,i}$.

Appendix B

Matrix/Fracture Transfer

The matrix/fracture transfer term is given by:

$$\frac{\hat{\tau}_{amf}}{B_\alpha} = -k_m \left(\frac{k_r}{\mu B} \right)_{amf} \sigma \left[\nabla p_{amf} - \nabla (\gamma_\alpha D)_{mf} \right] \quad (29)$$

Multiplying by $(V_R)_{ijk}$, including the conversion constant, and rearranging, the above equation is written as:

$$\begin{aligned} (\hat{\tau}_{amf})_{ijk} = & -0.001127 \left(k_m \sigma V_R \right)_{ijk} \left(\frac{k_r}{\mu B} \right)_{amf} \\ & \cdot \left[\nabla p_{amf} - \nabla (\gamma_\alpha D)_{mf} \right]_{ijk} \end{aligned} \quad (30)$$

By introducing the matrix/fracture transmissibility coefficient and realizing that the matrix/fracture potential term is expanded as follows:

$$\left[\nabla p_{amf} - \nabla (\gamma_\alpha D)_{mf} \right]_{ijk} = \left[(p_\alpha - \gamma_\alpha D)_f - (p_\alpha - \gamma_\alpha D)_m \right]_{ijk} \quad (31)$$

then, the above equation, for time level $n+1$, is written as:

$$(\hat{\tau}_{amf})_{ijk}^{n+1} = -T_{am,ijk}^{n+1} \left[(p_\alpha^{n+1} - \gamma_\alpha^{n+1} D)_f - (p_\alpha^{n+1} - \gamma_\alpha^{n+1} D)_m \right]_{ijk} \quad (32)$$

Appendix C

Matrix/Fracture Transmissibilities

The matrix/fracture transmissibility is written as:

$$T_{am,ijk}^{n+1} = 0.001127 \left(k_m \sigma V_R \right)_{ijk} \left(\frac{k_r}{\mu B} \right)_{amf,ijk}^{n+1} \quad (33)$$

σ is the matrix block's shape factor. For single-phase flow applications, Warren and Root have derived the following equation for σ :

$$\sigma = \frac{4N(N+2)}{L^2} \quad (34)$$

Where N is the number of normal sets of fractures: 1, 2, or 3, and L is the matrix block side length.

$$\left(\frac{k_r}{\mu B} \right)_{amf,ijk}^{n+1} = \left[(1 - \omega_{am}) \left(\frac{k_r}{\mu B} \right)_{af}^{n+1} + \omega_{am} \left(\frac{k_r}{\mu B} \right)_{am}^{n+1} \right] \quad (35)$$

The weighting factor ω has two values, either 1 or 0, depending on the potential difference between matrix and fracture. Thus:

$$\begin{aligned} \omega_{am,ijk} &= 1 \\ \text{if } (p_{am}^{n+1} - \gamma_{am}^{n+1} D_m)_{ijk} &\geq (p_{af}^{n+1} - \gamma_{af}^{n+1} D_f)_{ijk} \\ \omega_{am,ijk} &= 0 \quad \text{otherwise} \end{aligned} \quad (36)$$

$$\gamma_{am}^{n+1} = \frac{\rho_{am}^{n+1}}{144}$$

$$\gamma_{af}^{n+1} = \frac{\rho_{af}^{n+1}}{144}$$

Appendix D

Rate Calculations

For rate-control well boundary condition and full-implicit formulation, the wellbore rate equation is given by equation (3), where:

$(Q_T)_i$ = wellbore total flow rate, STB/DAY,

KK = number of perforated layers,

$(q_a)_{ik}$ = node ik phase flow rate; i.e.

$(q_w)_{ik} = WWP_{ik} (p_{bh} - p_{node})_{ik}$,

$(q_o)_{ik} = WOP_{ik} (p_{bh} - p_{node})_{ik}$,

$(q_L)_{ik} = (q_w)_{ik} + (q_o)_{ik}$,

$$WWP_{ik} = \left\{ WI \left(\frac{k_{rw}}{\mu_w} + \beta \frac{k_{ro}}{\mu_o} \right) \frac{1}{B_{rf}} \right\}_{ik} \quad (37)$$

$$WOP_{ik} = \left\{ WI (1 - \beta) \left(\frac{k_{ro}}{\mu_o B_o} \right)_f \right\}_{ik} \quad (38)$$

$$WI_{ik} = \left\{ \frac{0.001127 (2 \pi) \Delta z (\bar{k} \phi)_f}{\ln \frac{r_1}{r_w} + s} \right\}_{ik} \quad (39)$$

$$\bar{k}_{ik} = \left\{ \sqrt{k_x k_y} \right\}_{ik} \quad (40)$$

$$(r_1)_{ik} = \left\{ \frac{e^{-\gamma}}{2} \cdot \frac{\left[\left(\frac{k_y}{k_x} \right)^{\frac{1}{2}} \frac{A_{CV}}{\alpha} + \left(\frac{k_x}{k_y} \right)^{\frac{1}{2}} \alpha A_{CV} \right]^{\frac{1}{2}}}{\left(\frac{k_y}{k_x} \right)^{\frac{1}{4}} + \left(\frac{k_x}{k_y} \right)^{\frac{1}{4}}} \right\}_{ik} \quad (41)$$

where α is the control volume aspect ratio,

$$\beta_{ik} = \left\{ \begin{array}{ll} 1 & \text{for injection} \\ 0 & \text{for production} \end{array} \right\} \quad (42)$$

γ is the Euler constant = 0.57721566..., and

$c_\alpha \frac{\partial p_{bh}}{\partial t}$ is the wellbore storage.

Table 1: Reservoir properties for the five-spot pattern example

Reservoir dip angle, deg.	0
Drainage area, acres	8.265
Reservoir thickness, ft.	30
Matrix blocks shape factors	0.08
Fracture porosity	0.01
Matrix porosity	0.19
Fracture compressibility, 1/psi	3×10^{-6}
Matrix compressibility, 1/psi	3×10^{-6}
Fracture permeability, md	10,000
Matrix permeability, md	1
Fracture connate water saturation	0
Fracture residual oil saturation	0
Matrix connate water saturation	0.25
Matrix residual oil saturation	0.30
Pressure at WOC, psia	3959.89
Water FVF, B_w , RB/STB	1
Oil FVF, B_o , RB/STB	1
Water density, lb/ft ³	64
Oil density, lb/ft ³	52
Water compressibility, 1/psi	3.03×10^{-6}
Oil compressibility, 1/psi	1.50×10^{-6}
Water viscosity, cp	0.5
Oil viscosity, cp	2

Table 2: Reservoir properties for the five-well fractured reservoir

Reservoir dip angle, deg.	5
Drainage area, acres	373.049
Reservoir thickness, ft.	90
Matrix blocks shape factors	0.0008
Fracture porosity	0.01
Matrix porosity	0.20
Fracture compressibility, 1/psi	3×10^{-6}
Matrix compressibility, 1/psi	3×10^{-6}
Fracture permeability, md	21,000
Matrix permeability, md	1
Fracture connate water saturation	0
Fracture residual oil saturation	0
Matrix connate water saturation	0.25
Matrix residual oil saturation	0.30
Pressure at WOC, psia	3983.75
Water FVF, RB/STB	1
Oil FVF, RB/STB	1
Water density, lb/ft ³	63.14
Oil density, lb/ft ³	52
Water compressibility, 1/psi	3.03×10^{-6}
Oil compressibility, 1/psi	1.50×10^{-6}
Water viscosity, cp	0.5
Oil viscosity, cp	2

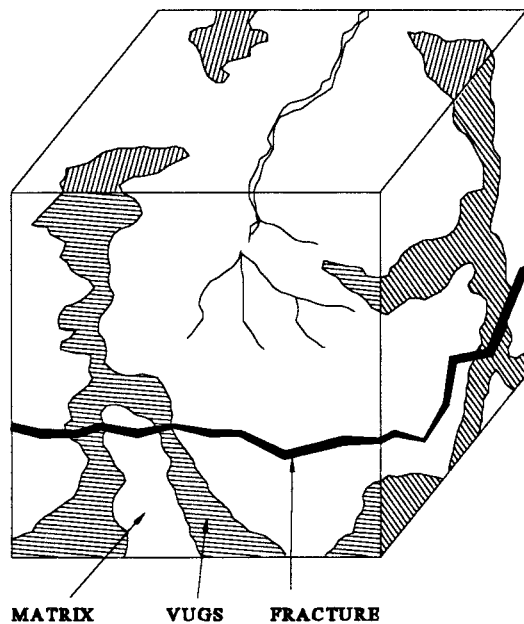


Fig. 1: Idealization of heterogeneous porous media: Actual reservoir (after Kazemi et al.)

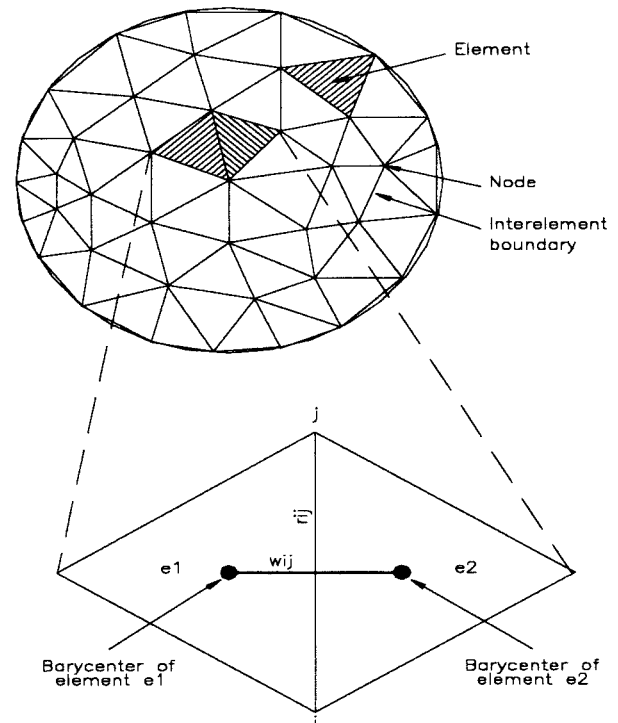


Fig. 3: Typical CVFE mesh for a two-dimensional domain using linearly interpolated triangular elements

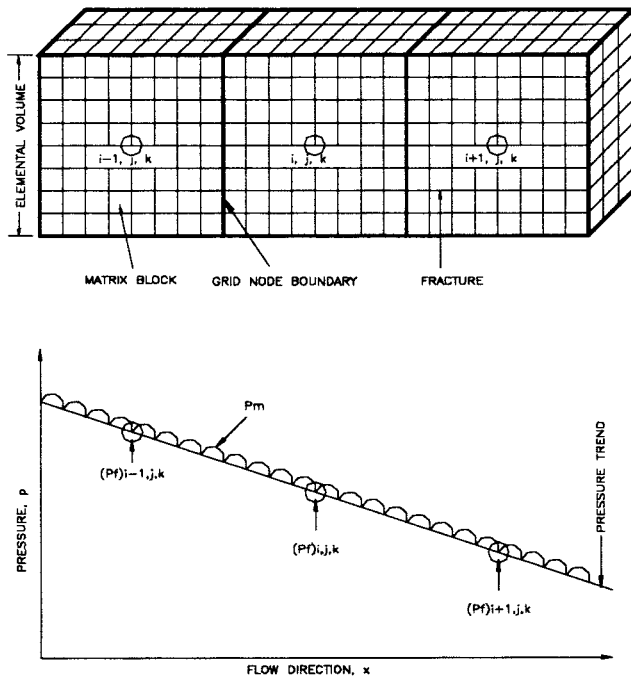


Fig. 2: Idealization of flow and elemental reservoir volumes containing matrix blocks (after Kazemi et al.)

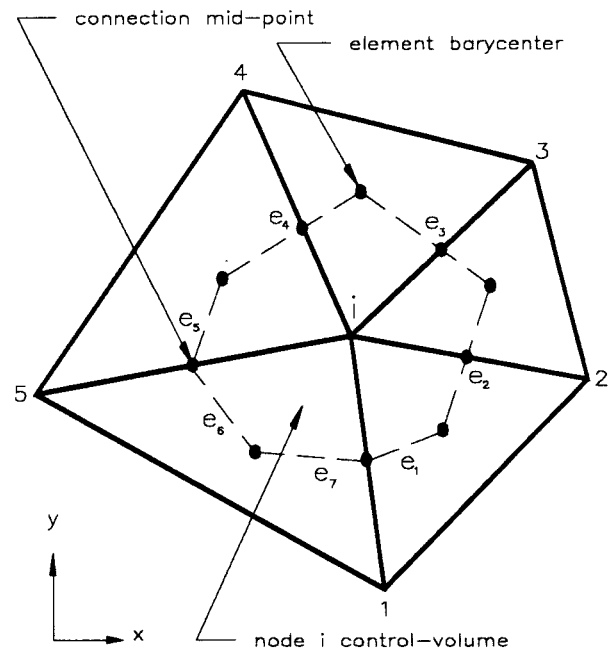


Fig. 4: Schematic of a control-volume surrounding node i

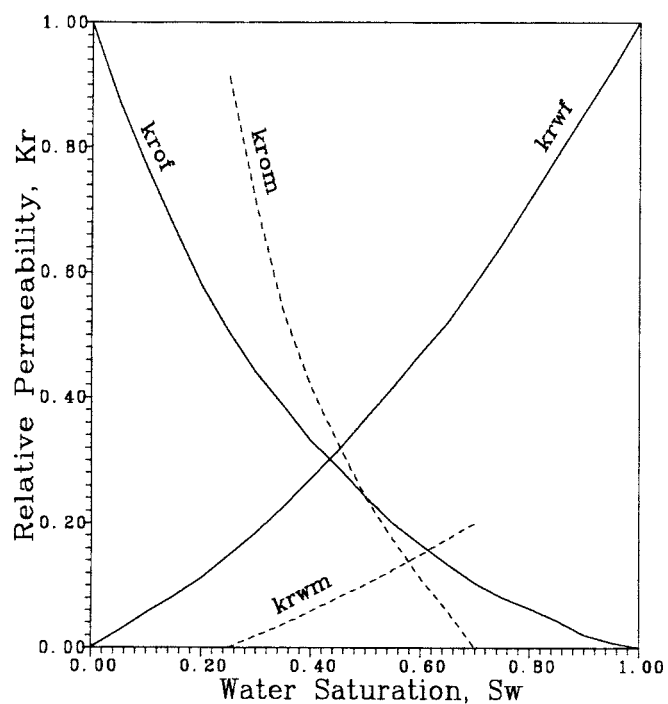


Fig. 5: Relative permeability data for the two examples

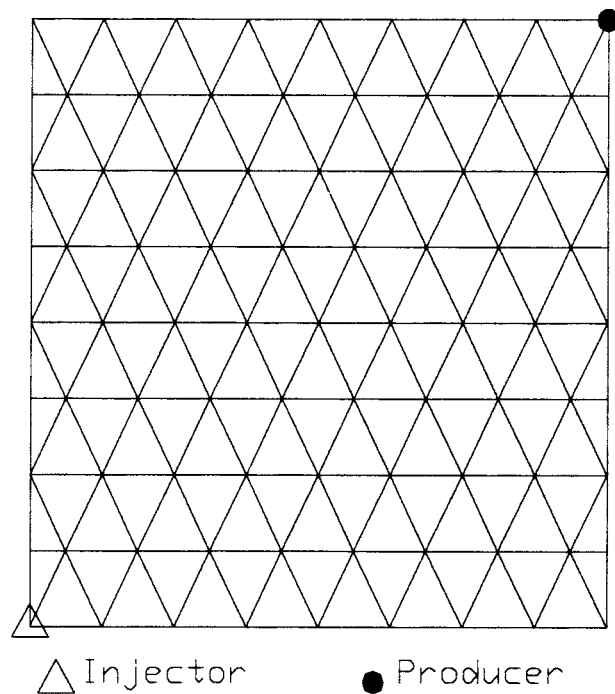


Fig. 7: The V8x8 CVFE mesh (600x600 ft, 136 elements)

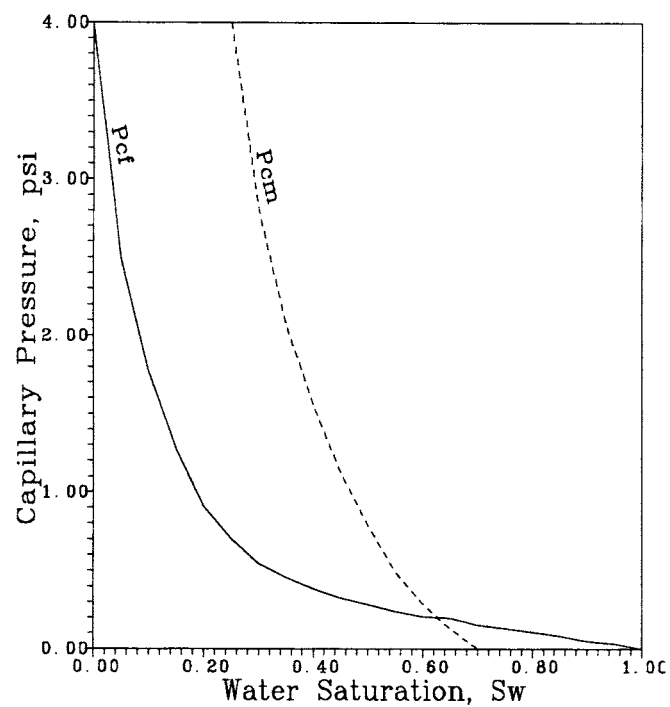


Fig. 6: Capillary pressure data for the two examples

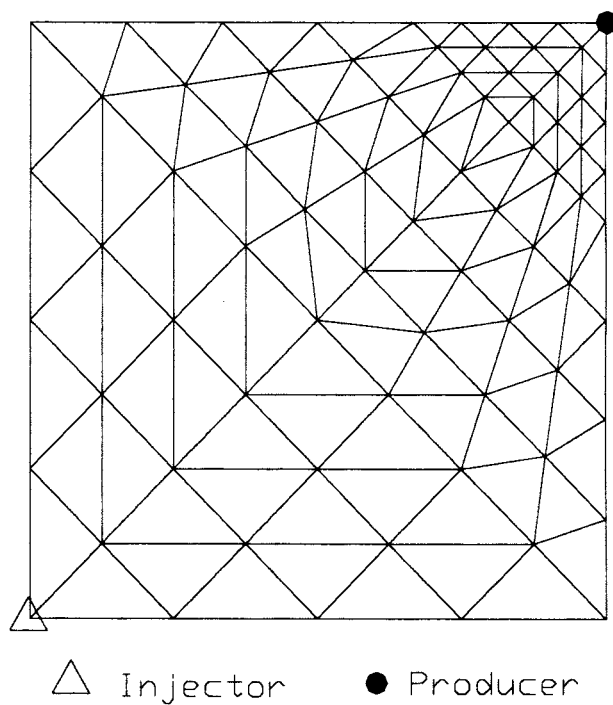


Fig. 8: The GT1 CVFE mesh (600x600 ft, 128 elements)

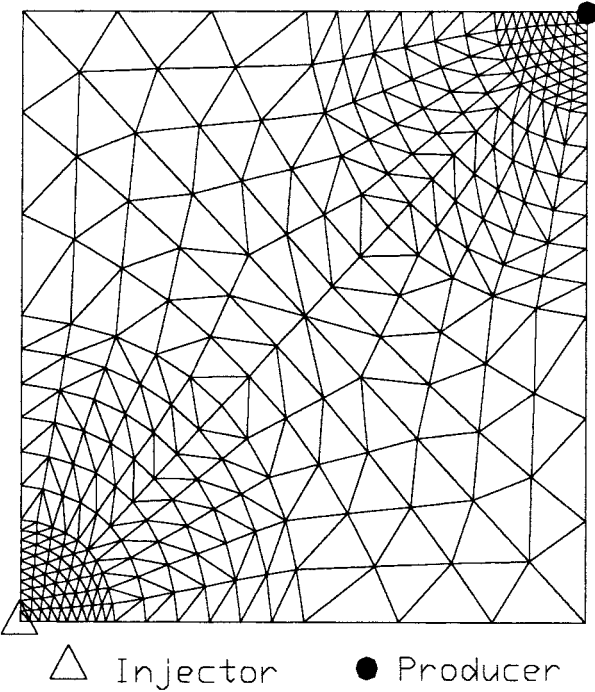


Fig. 9: The GT4 CVFE mesh (600x600 ft, 560 elements)

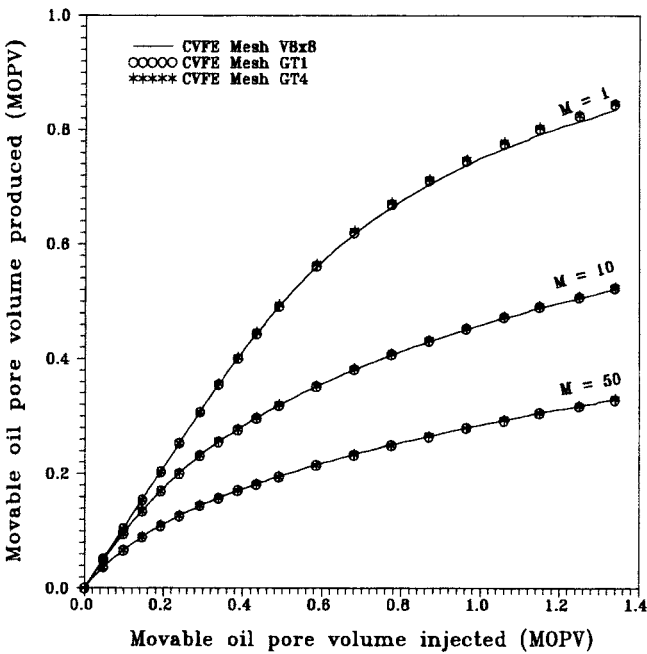


Fig. 11: Comparison between results obtained with the V8x8, GT1, and GT4 CVFE meshes

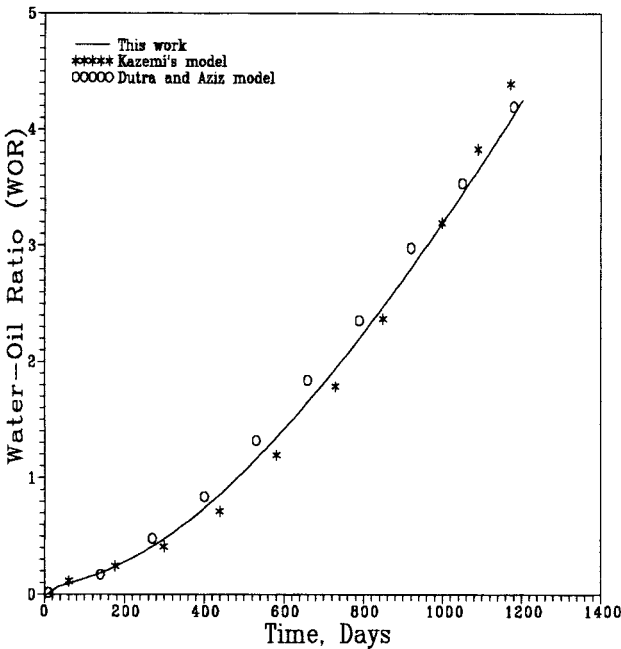


Fig. 10: Comparison between calculated water/oil ratios

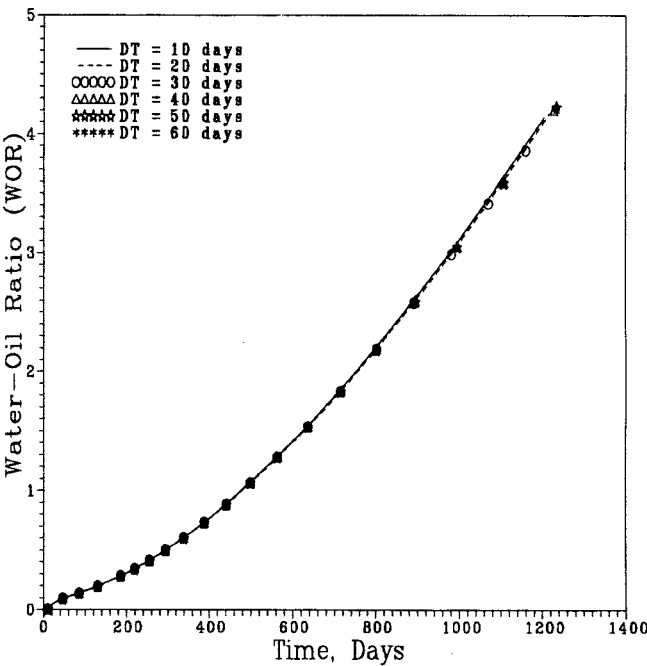


Fig. 12: Comparison of water/oil ratios for different timestep sizes

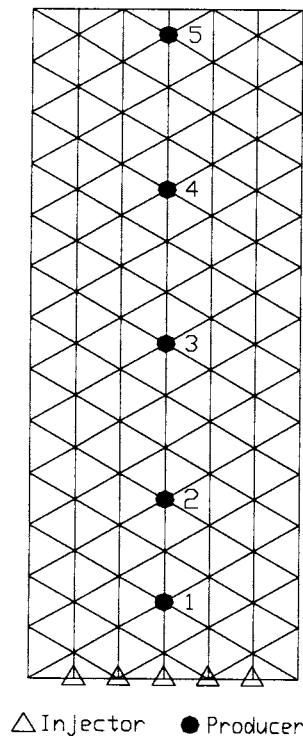


Fig. 13: The V13x6 CVFE mesh (6500x2500 ft, 162 elements)

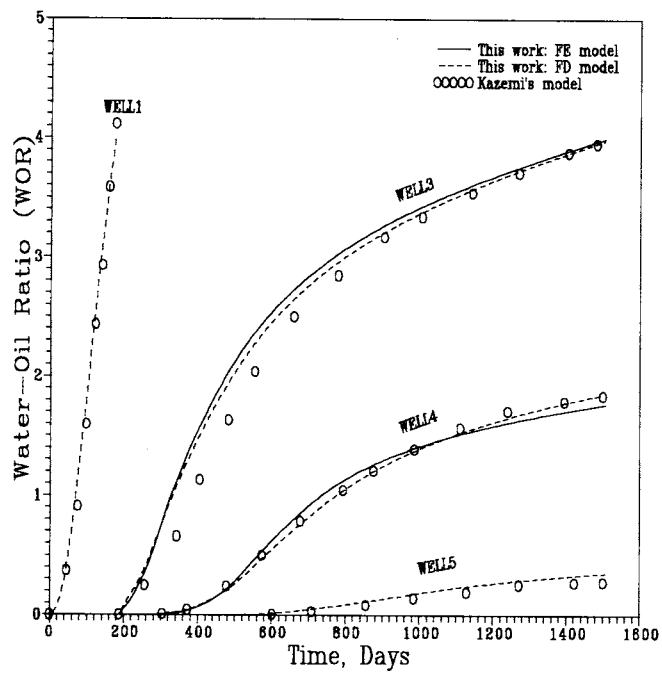


Fig. 14: Comparison between calculated water/oil ratios